A Stochastic Sigma Model For GLONASS Satellite Pseudorange

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BIOGRAPHY

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ABSTRACT

The GLONASS (Global Navigation Satellite System) is a satellite positioning system able to provide various number of air, marine, and any other type of users with all-weather three-dimensional positioning, velocity measuring and timing anywhere in the world or near-earth space.
As known, a GLONASS receiver performs passive measurements of pseudoranges and pseudorange rate of at least four GLONASS satellites as well as receives and processes navigation messages contained within navigation signals of the satellites. The navigation message supplies the satellites position both in space and in time. Combined processing of the measurements and the navigation messages of the four (three) GLONASS satellites allows user to determine three (two) position coordinates, three (two) velocity vector constituents, and to refer user time scale to the national reference time UTC(SU).
The purpose of this work is to define a stochastic model for pseudorange variances of GLONASS satellites able to provide its estimation. This evaluation is made for all satellites as a function of the elevation, independently of the user position, starting from real data. To achieve this goal a suitable software tool MATLAB® is developed. The tool is able to create a GLONASS sky from the broadcast ephemeris, to compute the pseudorange error and to process it.
The used data are extracted from observation and navigation RINEX files (containing both GPS and GLONASS measures)
From the known receiver position and the computed satellite coordinates, the geometric range is obtained

and compared with the pseudorange measurement, in order to achieve the pseudorange variance and build the model.
In order to validate the sigma stationary stochastic model, the results are compared with further data obtained from different stations.
The purpose of this work is the creation of a σ model particularly adapted in application as personal navigation device, characterized by the need of a real-time positioning and by a low computational power. The accuracy in real-time positioning can be improved, using a weighted least square (WLS) method for GNSS (GPS, GLONASS and in the future GALILEO or other feasible systems) measurements.
For GPS measurements, several suitable σ models for the WLS implementation are already in use; for GLONASS (or GLONASS-GPS together) the same is not available. So the need of studies about this topic.

1. INTRODUCTION

The satellite positioning systems are extensively used in very wide application area. For this reason many research programmes nowadays are focused on improving the satellite constellations (GALILEO) and/or modernizing the existing one (GPS).
But in order to achieve enhanced precisions also “integration” between the different systems and their augmentation are improved.
The constellations currently in operation are: the GPS and GLONASS. In recent years the latter was concerned from a program of constellation modernization and enhancement. So the need to start a statistical study of pseudorange measures is increased, above all in who is interested in all kinematic positioning applications.
[2, 3] The least-squares estimation technique is usually employed in the data processing step, and basically requires the definition of two models:
(a) the functional model, and (b) the stochastic model. The functional model describes the mathematical relationship between the GPS observations and the unknown parameters, while the stochastic model describes the statistical characteristics of the GPS observations (see [4] Leick (1995) and [11] Rizos etal (1997)).
The stochastic model is therefore dependent on the selection of the functional model.
A double-differencing technique is commonly used for constructing the functional model as it can eliminate many of the troublesome GPS biases, such as the atmospheric biases, the receiver and satellite clock biases, and so on.
In this paper a different approach is used. It is started from the pseudorange measurement for single-point positioning and, hence every error source is considered in its functional model.
[5] A GPS observation carries information on the geometric range between satellite and receiver (position) and on the clocks in use (time). In order to extract the desired position and/or time information by processing the collected data using a least-squares algorithm, a mathematical model has to be formulated.
Most simply the Satellite range observable can be split into a ‘signal’ part and a noise one:

$$\bar{p}(t) = \tilde{\rho}(t) + \bar{e}(t) \quad (1\text{-}1)$$

where the observable $\bar{p}$ is a stochastic variable, that is time dependent $\bar{p}(t)$ and hence a stochastic process. The deterministic signal part $\tilde{\rho}(t)$ contains the geometric range and systematic effects as for instance signal delay terms. The composition of satellites, signals, propagation media and receiver constitutes a dynamic measurement system. A more involved type of formulation, incorporating also the dynamics of the system, in terms of differential equations is given in e.g. [1] Eissfeller (1997). In practice the above equation is considered instead at discrete epochs.
Many researchers have emphasized the importance of the stochastic model, especially for high accuracy applications, for example, [7] Barnes et al. (1998), [8] Cross et al. (1994), [9] Han (1997), [10] Teunissen (1997), [12] Wang (1998), [13] Wang et. al. (2001) for both the static and kinematic positioning applications. In principle it is possible to improve the accuracy and reliability of GPS results through an enhancement of the stochastic model.
Previous studies, based on GPS observables, have shown that measurements have a heteroscedastic, space- and time- correlated error structure ([12] Wang (1998), [14] Wang et al. (1998)). The challenge is to find a way to realistically incorporate such information into the stochastic model.
This paper deals only with the static single-epoch positioning case and the above mentioned stochastic model is assumed as hypothesis of the measurement errors. In the first part of this work, all considerations on which this assumption is based are showed; while in the second one, the studied model is introduced.

2. GLONASS OVERVIEW

GLONASS (Global Navigation Satellite System) is a satellite positioning system able to provide various number of air, marine, and any other type of users with all-weather three-dimensional positioning, velocity measuring and timing anywhere in the world or near-earth space.
GLONASS is the Russian alter-ego of the American GPS; its development began in 1976 and the system became complete in 1995. Successively, the economical crisis of Russia caused a rapid disrepair of the system, but in 2001 a restoring phase started with the goal to make the system completely operational on global basis in 2009.
GLONASS navigation system is based on the concept of “one-way ranging”, in which the unknown user position is obtained measuring the time of flight of signals broadcasted by satellites at known positions and times.

GLONASS is made up of three components: a Space Segment, a Ground Segment and a User Segment.
The Space segment nominally consists of 24 satellites, orbiting in three planes whose ascending nodes are 120° apart. Eight satellites are equally spaced on each plane. The phase displacement between the planes is 15°. The orbits are theoretically circular, with a period of 11^h15^m, corresponding to an approximate altitude of 19100 km. The nominal orbital inclination is 64.8°. Such a constellation guarantees a continuous and global coverage of Earth surface and the near-earth space.
At present (July 2009) 20 satellites are in orbit, with 18 operational and 2 in maintenance.
The Ground Segment consists of the System Control Centre and a network of Command and Tracking Stations, all deployed on the Russian territory.
The User Segment includes all the receivers able to process GLONASS signals to navigation purpose, without limitations of number, type (civil or military) or goal (air, marine, space or terrestrial navigation).
Each GLONASS satellite transmits a different carrier on L band, modulated by a ranging code, used to measure the distance between satellite and receiver, and a navigation message, storing navigation information as satellite position and health.
A GLONASS receiver performs passive measurements of pseudoranges and pseudorange rate of at least four GLONASS satellites as well as receives and processes navigation messages contained within satellite signals. Combined processing of the measurements and the navigation messages of the at least four GLONASS satellites allows user to determine three position coordinates, three velocity vector constituents, and to refer user time scale to the national reference time UTC(SU).
GLONASS and GPS systems are similar, but there are some fundamental differences.
The systems operate with different time references: GPS time is related with UTC(USNO), Coordinated Universal Time as maintained at the United States Naval Observatory, and GLONASS time is related to UTC(SU), UTC as maintained by Russia. The offset between the two time references can be calibrated, but this information is not included in the navigation messages broadcasted by the satellites; also the RINEX version at the moment in use (version 2.10) does not provide the UTC(USNO)-UTC(SU) offset. The problem will be overcome with the next generation of satellites, that are planned to broadcast the offset between the time scales and with the next released RINEX version (3.0).
This offset can be included as a further navigation unknown variable.
GPS and GLONASS express the positions in different Geodetic Coordinate Frames. GPS uses WGS84, GLONASS uses PZ-90. The two Datum are nearly coincident and are linked by a well-defined mathematical transformation.
GPS and GLONASS transmit signals with different features. The signal bandwidths are different, affecting slightly the measurement noise. The two systems use different multiple access schemes: GLONASS uses Frequency Division Multiple Access (FDMA), with different satellites transmitting the same pseudorandom noise code but at different carrier frequencies and GPS uses CDMA (Code Division Multiple Access), with different codes transmitted at the same frequency [16].
The integration GPS-GLONASS provides additional ranging sources, improving the positioning service availability; the use of a multi-constellation could make the positioning possible also in conditions of degraded visibility, like urban canyon and mountainous areas, or in critical application, as air navigation. However the combination GPS-GLONASS requires expedients for adapting the models of measurement errors to account the differences between the two systems.

3. STATISTICAL BACKGROUND

Within the navigation system GLONASS, the block used in signal transmission can be seen as a communications system. The theory of probability and stochastic processes is an essential mathematical tool in the study of communication systems. Many of the random phenomena that occur in nature are function of time: also the thermal noise that is generated on the resistance of electronic devices, such as a GLONASS receiver measurement can be seen as a time function.
Following, it is introduced a brief overview on a stochastic process.
Instead of dealing with only one possible "reality" of how the process might evolve under time (as is the case, for example, of solutions of a differential equations) in a stochastic or random process there is some indeterminacy in its future evolution described by probability distributions. This means that even if the initial condition (or starting point) is known, there are many possibilities the process might go to, but some paths are more probable than others.
So at any given epoch, the value of a stochastic process, whether it is the value of the noise voltage generated by a resistor or the amplitude of a signal received by a GLONASS receiver is a random variable. Thus, we may view a stochastic process as a random variable indexed by time parameter t. The noise voltage generated by a single resistor represents a single realization of the stochastic process [17,18].
Like other GNSS systems, GLONASS data processing is normally carried out using the least-squares method. The implementation of the least-squares method requires a set up of both functional and stochastic models. GLONASS measurements are usually assumed to have the same precision and to be statistically independent in time and space. This is strictly not true an so this assumption has to be justified.

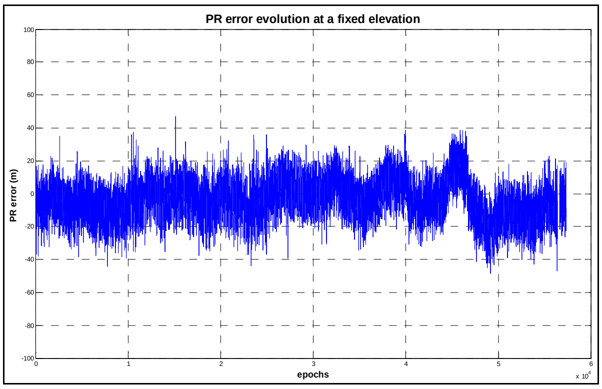

Figure 3-1: *Pseudorange error time evolution at a fixed elevation*

In Figure 3-1 is shown the evolution over time of the pseudorange error, when the satellite passes to a fixed elevation. It seems clear that the evolution of the expectation value does not depend on time but remains confined to a constant range.

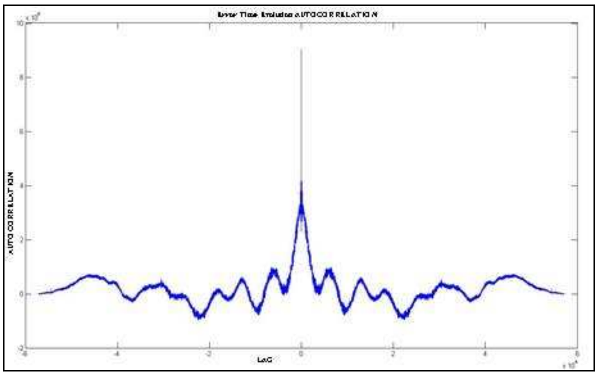

Figure 3-2: *Error time evolution autocorrelation*

In Figure 3-2 the evolution of the autocorrelation function for the same data set of Figure 3-1: *Pseudorange error time evolution at a fixed elevation* is shown. As it can be seen, the function assumes the maximum in the axes origin and it is an even function. Moreover the function tends to take values increasingly smaller with rising delay; this means that with increasing distance in time, samples can be regarded as independent.

Different types of independent variables can be used as indicators of quality for the creation of a more realistic stochastic model; more adopted criterions are based on satellite elevation, Signal-to-Noise Ratio (SNR) and least-squares residual [15].

4. DETERMINISTIC COMPUTATION OF PSEUDORANGE ERROR AND SIGMA MODEL BUILDING

The pseudorange error computation is the core of this work; to analyze the pseudorange error statistical features it is necessary to remove all the deterministic effects on it.

The raw pseudorange error is defined as:

$$\Delta = \rho - d \qquad (4\text{-}1)$$

with ρ unprocessed pseudorange measurement and d geometric distance receiver-satellite.
The error (4-1) is not directly usable, but it has to be corrected applying suitable deterministic models.
The input for the pseudorange error computation are stored in three types of RINEX files:

- observation data (mixed)
- navigation message (GPS)
- navigation message (GLONASS)

The raw pseudorange ρ (from RINEX observation files) is corrected for satellite and receiver clock errors according to the equation (4-2):

$$\rho_{CORR} = \rho + c \cdot dt_{SAT} - c \cdot dT \qquad (4\text{-}2)$$

where:
ρ_{corr} corrected pseudorange measurement;
dt_{sat} satellite clock bias;
dT receiver clock bias and c light speed.

The GLONASS satellite clock bias is estimated using the correction model (4-3):

$$dt_{SAT} = -\tau_N + \gamma_N \cdot (t_{SAT} - t_b) - \tau_C \qquad (4\text{-}3)$$

where:
t_b is the reference time of satellite ephemeris data;
τ_N is the offset between satellite and GLONASS system time scale at t_b;
γ_N is the relative deviation of the predicted carrier frequency value of the considered satellite from nominal value at t_b;
t_{sat} is the time for which GLONASS time is desired;
τ_C is the GLONASS time scale correction to UTC(SU) time.

Strictly t_{sat} should be referred to system time scale, but for practical purpose a rough t_{sat} is used, given in satellite scale. The parameters τ_N, γ_N, t_b are immediate information and are stored in the data section of RINEX GLONASS navigation file; τ_C is a non-immediate information and is obtained from the header of the same type of file.
The receiver clock bias cdT is the fourth unknown of the GPS navigation equation (4-4):

$$\boldsymbol{\Delta\rho} = \mathrm{H} \cdot \boldsymbol{\Delta x} + \boldsymbol{\varepsilon} \qquad (4\text{-}4)$$

where:
$\boldsymbol{\Delta\rho}$ (nx1) is the measurement vector compensated by a priori information;
H (nx4) the geometry matrix;
$\boldsymbol{\Delta x}$ (4x1) the unknown vector of corrections from a priori to updated state;

ε (nx1) measurement errors vector (considered Gaussian);
n number of observed satellites.

The set of equation is solved for Δx by means of least square method and the solution is given by equation (4-5):

$$\mathbf{\Delta\hat{x}} = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \mathbf{\Delta\rho} \qquad (4\text{-}5)$$

where R is the pseudorange error covariance (nxn) matrix.
For cdT computation, all pseudorange errors are considered independent with equal variances; so R is set equal to identity matrix and equation (4-5) becomes:

$$\mathbf{\Delta\hat{x}} = \left(H^T H\right)^{-1} H^T \mathbf{\Delta\rho} \qquad (4\text{-}6)$$

The corrected pseudorange error is defined as:

$$\Delta_{CORR} = \rho_{CORR} - d \qquad (4\text{-}7)$$

and for its computation (4-2), (4-3) and (4-6) are used.
In Figure 4-1: *Comparison between Δ and Δ_{corr}* the comparison between raw pseudorange error Δ and corrected pseudorange error Δ_{corr} for a single GLONASS satellite during about one hour is shown. The raw pseudorange error has the typical evolution as a saw-tooth wave, because the prevailing error is the receiver clock bias. Δ_{corr} is not constant, but varies in a few meters range; in the Figure 4-1: *Comparison between Δ and Δ_{corr}* this can not be appreciated due to the much wider range of Δ. A zoom of the comparison is shown in Figure 4-2, where the not constancy is clear.
Obviously raw pseudorange error can not be use directly in a stochastic analysis; the modified error Δ_{corr} instead can be considered a stationary series.

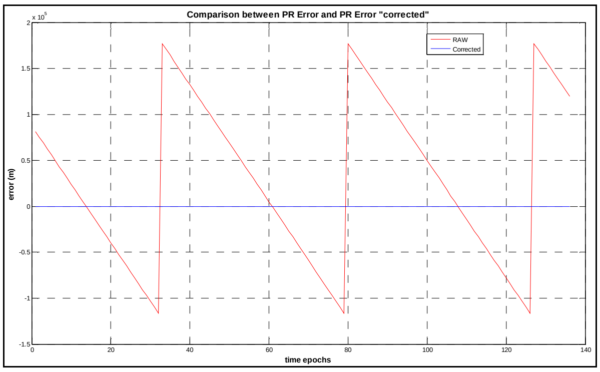

Figure 4-1: *Comparison between Δ and Δ_{corr}*

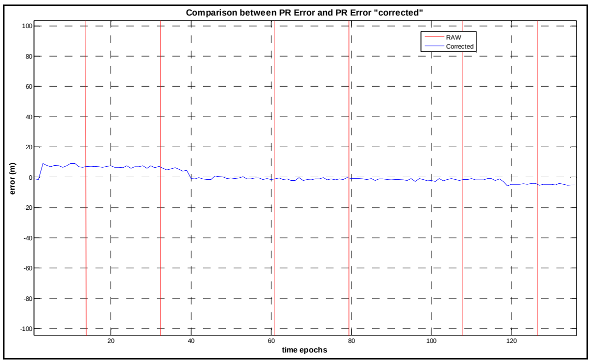

Figure 4-2: *Particular of comparison between Δ and Δ_{corr}*

The geometric distance receiver-satellite d in (4-7) is computed as:

$$d = \sqrt{\left(X_R - X_S\right)^2 + \left(Y_R - Y_S\right)^2 + \left(Y_R - Y_S\right)^2} \qquad (4\text{-}8)$$

where (X_R,Y_R,Z_R) are receiver coordinates and (X_S,Y_S,Z_S) satellite coordinates both in WGS84 reference system.
The receiver position is known precisely, because fixed station is considered; while the satellite coordinates are computed integrating the motion equation with 4^{th} order Runge-Kutta method, using initial conditions from GLONASS RINEX navigation file.

In Figure 4-3 the algorithm for the error computation is described in detail. For each observed satellite a pseudorange measurement PR is obtained at a reception epoch t_r; a raw transmission epoch has to be computed as (4-9) to select the appropriate correction parameters for the equation (4-3).

$$t_{tRAW} = t_r - PR/c \qquad (4\text{-}9)$$

The so obtained satellite clock bias is used for pseudorange correction (4-3) and for precise time transmission epoch:

$$t_t = t_{tRAW} - dt_{SAT} \qquad (4\text{-}10)$$

GLONASS satellite ephemerides are propagated up to epoch t_t and referred to WGS84 by algorithms described in appendix.
The time of flight TOF (or travel time) is define as the time interval for the signal to travel from the satellite to the receiver:

$$TOF = t_r - t_t \qquad (4\text{-}11)$$

The computed satellite WGS84 coordinates have to be corrected, by a $\omega_e \cdot TOF$ counter clockwise rotation to refer satellites and station coordinates to the same reference system.

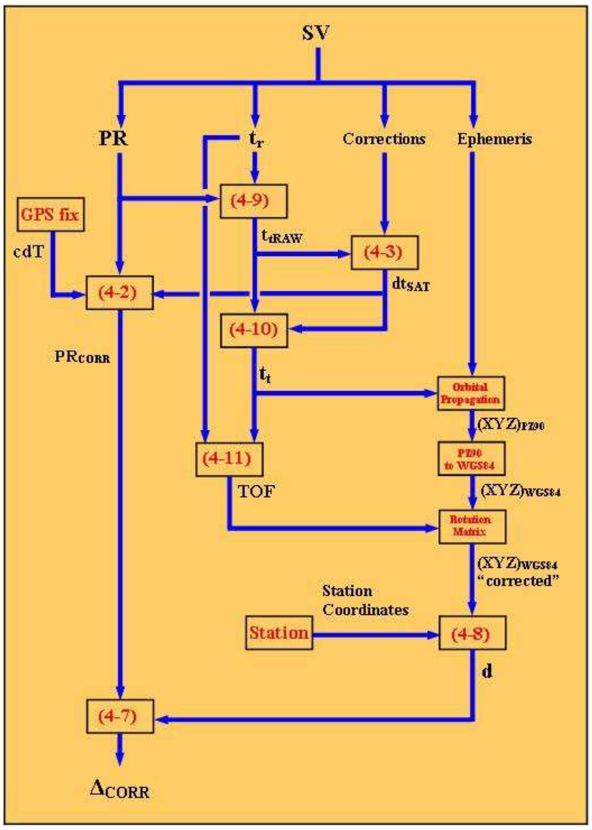

Figure 4-3: *Flow chart for pseudorange error computation*

The corrected pseudorange error is used to build the proposed sigma model. The stationary assumption, made on the stochastic process, makes possible to obtain an elevation dependent sigma model; GLONASS measurement error tends to be different for each satellite (due to their different clocks, ages, health histories, frequency, etc), but in this paper the pseudorange error long-term distribution is considered equivalent for each satellite.
On this basis measurement errors from all satellites at the same elevation are considered a realization of the same stochastic process and therefore they are processed together to achieve the standard deviation for the fixed elevation.

5. ANALYSIS AND RESULTS

The purpose of this work is to create a stochastic model of GLONASS pseudorange error, starting from a massive amount of real data.
A whole year of data are used, obtained from the permanent GNSS station of Sala Consilina, belonging to the GNSS Campania Regional Network.
The statistical analysis results of the considered data set are plotted in Figure 5-1.

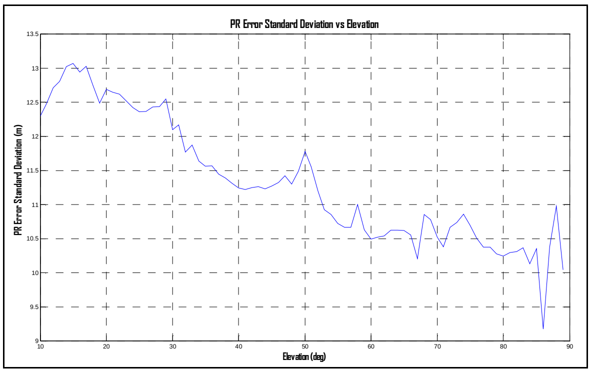

Figure 5-1: *Pseudorange error standard deviation versus elevation*

The above figure shows the distribution of pseudorange error standard deviation versus the satellite elevation. As expected the standard deviation trend decreases as satellite elevation raises. The diagram shape is irregular owing to the experimental basis of the used data.
In order to provide a more smoothed trend, a polynomial regression function is used for data fitting. After various trails, a fourth order polynomial with equation (5-1) resulted the best choice.

$$\sigma = \sigma(El) = p_1 \cdot El^4 + p_2 \cdot El^3 + p_3 \cdot El^2 + p_4 \cdot El + p_5 \quad (5\text{-}1)$$

where:
p_1=-4.8267e-7
p_2=1.0168e-4
p_3=0.0070
p_4=0.1410
p_5=11.9073

The results are shown in Figure 5-2.

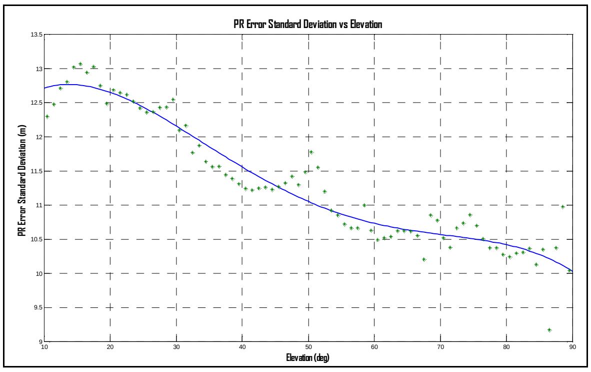

Figure 5-2: *Pseudorange error standard deviation and fitting function versus elevation*

In order to verify the proposed model a comparison is made using data downloaded from a different station placed in Ankara (belonging to IGS network). The pseudorange errors are computed by the algorithm above described and the belonging to different confidence intervals is verified. The results are shown in Figure 5-3.

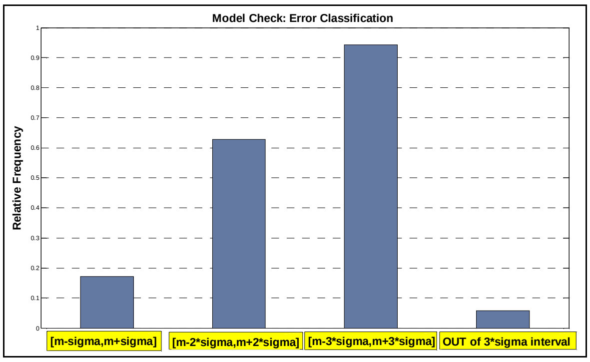

Figure 5-3: *σ model verification*

About 95% of errors fall through the [m-3σ, m+3σ] range, where m is the error mean value; this partially confirms the validity of the proposed model.

6. FUTURE WORK

The next steps of this research are further algorithm enhancements for a more accurate model: more biases can modeled, reduced or eliminated as UTC(USNO)-UTC(SU) offset.
The use of more stations, placed world-wide, can make the model more position independent.
The final goal is to employ the obtained model for a WLS solution.

7. APPENDIX: ALGORITHMS FOR RE-CALCULATION OF EPHEMERIS TO CURRENT TIME AND PZ90 TO WGS84 TRANSFORMATION

In order to achieve the goal of this paper it is necessary to "build" the GLONASS sky. For this purpose a MATLAB Software tool is developed in order to compute the GLONASS constellation satellites positions, starting from the broadcast ephemeris, stored in Navigation Message. The satellite coordinates were obtained from the integration of the following equations [1]:

$$dx/dt = Vx$$
$$dy/dt = Vy$$
$$dz/dt = Vz$$
$$dV_x/dt = -\frac{\mu}{r^3}x - \frac{3}{2}J_0^2\frac{\mu a_e^2}{r^5}x\left(1-\frac{5z^2}{r^2}\right) + \omega^2 x + 2\omega V_y + \ddot{x}$$
$$dV_y/dt = -\frac{\mu}{r^3}y - \frac{3}{2}J_0^2\frac{\mu a_e^2}{r^5}y\left(1-\frac{5z^2}{r^2}\right) + \omega^2 y + 2\omega V_y + \ddot{y}$$
$$dV_z/dt = -\frac{\mu}{r^3}z - \frac{3}{2}J_0^2\frac{\mu a_e^2}{r^5}z\left(1-\frac{5z^2}{r^2}\right) + \ddot{z}$$

(7-1)

where:
$r = \sqrt{x^2+y^2+z^2}$ is the distance satellite-Earth center;
$\mu = 398600,44*10^9\ m^3/s^2$ is the Gravitation constant;
$a_e = 6378136m$ is the Earth semi-major axis;
$J_0^2 = 1082625,7*10^{-9}$ is second zonal harmonic of the geopotential;
$\omega = 7,292115*10^{-5}$ is the Earth rotation rate.

Satellite position (x_0, y_0, z_0) and velocity $(\dot{x}_0, \dot{y}_0, \dot{z}_0)$ such as the accelerations $(\ddot{x}(t), \ddot{y}(t), \ddot{z}(t))$ due to luni-solar gravitational perturbation, are the initial conditions for integration and are extracted from the GLONASS Navigation file in RINEX format.

In order to trace the geometric distance between observer position (in WGS84 coordinate System) and Satellite (PZ90), there was the necessity to transform the PZ-90 coordinates in WGS84.
As defined a PZ-90 system is an Earth-Centered Earth-Fixed reference frame defined as follows:

- The origin is located at the center of the Earth's body;
- The Z-axis is directed to the Conventional Terrestrial Pole as recommended by the International Earth Rotation Service (IERS);
- The X-axis is directed to the point of intersection of the Earth's equatorial plane and the zero meridian established by BIH;
- The Y-axis completes the coordinate system to the right-handed one.

In order to carry out the PZ90 to WGS84 transformation a rotation, a translation and scale factor are implemented; the transformation parameters proposed in [20] are applied as shown in :

$$\begin{bmatrix}x\\y\\z\end{bmatrix}_{WGS84} = \begin{bmatrix}-0.47m\\-0.51m\\-1.56m\end{bmatrix} + (1+22\cdot10^{-9})\begin{bmatrix}1 & -1.728\cdot10^{-6} & -0.017\cdot10^{-6}\\ 1.728\cdot10^{-6} & 1 & 0.076\cdot10^{-6}\\ 0.017\cdot10^{-6} & -0.076\cdot10^{-6} & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}_{PZ90}$$

(7-2)

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