Stability Analysis of Strapdown Seeker Scale Factor Error and LOS Rate

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ABSTRACT

An inertial measurement unit (IMU) consisting of three gyroscopes and three accelerometers is utilized to carry out guidance performance in a short range ground to ground missile system. To improve the accuracy of trajectory, a seeker is applied and this seeker measurement is provided to a PNG law. A Strapdown seeker has advantages compared to a gimbaled seeker in terms of size and FOV limit. If the strapdown seeker output is combined with the output of an inertial sensor, the potential for instability exists if the scale factors and gain of the two sensors are not equal. Because the strapdown seeker provides LOS angles from body to target in a body frame, the LOS angles need to be changed to their rates in the inertial frame for PNG.

This paper presents the method of a LOS rate derivation from image plane values of the output of the strapdown seeker and analyzes the effects of the instability generated by the scale factor errors of the strapdown seeker and inertial sensor. The proposed derivation of LOS rate and analysis of the effect of scale factor error are verified by a simple but realistic navigation and guidance control simulation. When we assume the threshold of the miss-distance is within 10 meters, results are satisfied within the threshold with a 0.5% scale factor error. When we assume the threshold of the miss-distance is within 2 meters, results are satisfied within the threshold with a 0.45% scale factor error.
Through the simulation, we can find the effect of scale factor error, and this paper suggests further work towards a solution to this instability.

INTRODUCTION

Instead of increasing the quantity of missile warhead, the improvement of the accuracy in guidance equipment brings on the accuracy rate on target with respect to the striking power. There are seekers which can improve the accuracy of the missile. The seeker provides the LOS angle from missile to target through the pitch and yaw angle. The seeker consists of a gimbaled seeker and a strapdown seeker.

In conventional gimbaled structures, a seeker provides the LOS rate (Jacques Waldmann, 2002). Recent advancements in seeker technology have resulted in seeker designs with much larger FOV (fields-of-view) and seeker tracking characteristics which do not require the seeker centerline to point in the general vicinity of the target. Examples of such seekers include optical and radar correlators, holographic lenses used with laser detectors, and phased array antennas. The potential advantages of such seekers are numerous, stemming fundamentally from the fact that the seeker can now be rigidly fixed to weapon body.

These body-fixed strapdown seekers have the merit of eliminating the tracking rate limits and structural limitations pertinent to inertially stabilized gimbaled seekers. They also reduce the mechanical complexity of implementation and calibration. The elimination of mechanical moving parts would in turn eliminate frictional cross-coupling between pitch and yaw tracking channels, yielding the advantage of increased reliability of electronic components over mechanical ones. Finally, there is potentially a significant cost savings associated with eliminating the gimbals (Paul L. Vergez and James R. McClendon, 1982).

The strapdown seeker technology has been developed significantly over the last several years for air-to-surface and air-to-air applications (Raman K. Mehra and Ralph D. Ehrich, 1984). One such application is the LCPK (Low Cost Precision Kill) and the APKWS (Advanced Precision Kill Weapon System) in the USA.

The proportional navigation guidance law is frequently employed in short range missiles because it can easily be implemented and provides a near optimal guidance for constant velocity targets. The conceptual idea behind PN guidance is that the missile should keep a constant bearing to the target at all times, which will result in an eventual impact (Jing Xu, Kai-Yew Lum and Jian Xin Xu, 2007). Acceleration commands in missiles guided by proportional navigation require the measurement of a relative LOS rate between missile and target.

If the strapdown seeker output is combined with the output of an inertial sensor, the potential for instability exists if the scale factors and gain of the two sensors are not equal. Because the strapdown seeker provides LOS angles from body to target in a body frame, for PNG, the LOS angles need to be changed to their rates in the inertial frame.

This paper presents a method of LOS angle and LOS rate derivation from the image plane values, which are the output of the strapdown seeker. This paper also analyzes the effects of the instability generated by the scale factor errors of the strapdown seeker and inertial sensor. In order to evaluate the derived LOS angle and rate and to analyze the scale factor error, a short range missile employing a PNG law is simulated using the guidance control navigation package simulation. The simulation results show that the method of LOS angle and LOS rate derivation is accurate. Moreover, when we assume the threshold of the miss-distance is within 2 meters, results are satisfied within the threshold with a 0.45% scale factor error.

DERIVATION OF LOS ANGLE AND LOS RATE IN STRAPDOWN SEEKER MISSILE

The PNG law needs the line of sight rate with respect to the inertial frame in order to perform guidance. In this section, the strapdown seeker is assumed to measure the LOS angle in the body frame. To calculate the PNG law, the LOS vector \( \vec{L} \) with respect to inertial frame is defined as follows.

\[
\vec{L} = \vec{P}_t - \vec{P}_v = \begin{bmatrix} L'_x & L'_y & L'_z \end{bmatrix}^T
\]

The LOS angles \( \psi_{LOS} \) and \( \theta_{LOS} \) are defined in the inertial frame. Figure 1 shows the LOS angles in the inertial frame.

\[
\psi_{LOS} = \tan^{-1}\left(\frac{L'_y}{L'_z}\right)
\]

Figure 1. LOS angles in inertial frame
The image plane value \([y, z]\) is derived as follows (Joongsup Yun, Chang-Kyung Ryoo and Taek-Lyul Song, 2008).

\[
\theta_{\text{LOS}} = \tan^{-1} \left( \frac{L'_y}{\sqrt{(L'_x)^2 + (L'_z)^2}} \right)
\]

(2-b)

The LOS vector \(\vec{L}'\) in inertial frame can be changed into LOS vector \(\vec{L}^b\) in body frame through a coordinate transformation matrix \(C^b_i\). The LOS angles \(\psi_{\text{LOS}}, \theta_{\text{LOS}}\) are defined in the body frame.

The target angular position in the image plane is measured by the sensor. Figure 2 shows the LOS angles in the body frame and target angular position in the image plane.

Figure 2. Image plane

\[
\vec{L}^b = C^b_i \vec{L}' = \begin{bmatrix} L^b_x & L^b_y & L^b_z \end{bmatrix}
\]

(3)

\[
\theta_{\text{LOS}} = \tan^{-1} \left( \frac{L^b_y}{L^b_z} \right)
\]

(4-a)

\[
\psi_{\text{LOS}} = \tan^{-1} \left( \frac{L^b_y}{\sqrt{(L^b_x)^2 + (L^b_z)^2}} \right)
\]

(4-b)

where \(C^b_i\) denotes the coordinate transformation matrix from inertial frame to body frame.

\(C_z(\phi)\) denotes the coordinate transformation matrix representing the rotation of angle \(\phi\) about the x-axis.

\(\phi, \theta, \psi\) are the vehicle’s roll, pitch, and yaw attitudes.

\[
C_z(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & S\phi \\ 0 & -S\phi & C\phi \end{bmatrix}
\]

\(S\phi = \sin \phi, C\phi = \cos \phi\)

In the image plane, the image plane value \([y, z]\) is derived as follows (Joongsup Yun, Chang-Kyung Ryoo and Taek-Lyul Song, 2008).

\[
Z = l \tan \theta_{\text{LOS}}
\]

(5-a)

\[
Y = \sqrt{Z^2 + I^2} \tan \psi_{\text{LOS}}
\]

(5-b)

From the image plane value which is the measurement of strapdown seeker, we can derive the LOS angles with respect to the inertial frame through equations 1 - 5. The notation used in Figure 2 is summarized as follows:

- \(\vec{L}^b\): LOS vector in body frame
- \(l\): lens focal length
- \(x^b, y^b, z^b\): x-axis, y-axis and z-axis in body frame
- \([y, z]\): Image plane value
- \(\psi_{\text{LOS}}, \theta_{\text{LOS}}\): LOS yaw angle represented in body frame

In order to examine the equations, we performed a simulation using two simple reference trajectories.

The assumptions of the first simulation are as follows.

The vehicle’s initial attitude is roll 0deg, pitch 20deg and yaw 0deg. The vehicle’s final attitude is roll 0deg, pitch -20deg, yaw 0deg. The total flight time is 15seconds and vehicle speed is 200m/s. The target position is fixed and the strapdown seeker FOV limit is ±40deg. Normal lens focal length is 50[mm]. Figure 3(a) shows the reference trajectory and the green square represents the initial position, and the red circles represent points at every 2 seconds.

As shown in Figure 3(a), the vehicle flies to the north direction, so the path of the image plane value is headed north. Figure 3(b) shows the image plane value. From this value, we can derive the LOS angle in inertial frame and body frame which are showed in Figure 3(c).

In the second simulation, the assumptions are the same as the first simulation except for the vehicle’s initial attitude yaw 30deg and final yaw 30deg. Figure 4(a) shows the reference trajectory and Figure 4(b) shows the image plane value.

As shown in Figure 4(a), the vehicle flies without changing the heading, so the path of the image plane value is the north direction.
From this value, we can also derive the LOS angle in the inertial frame and body frame which are shown in Figure 4(c). The initial $\psi$ is 30deg and the initial $\psi_{LOS}$ is 0deg because the vehicle’s initial attitude is yaw 30deg and final is 30deg. In this simulation, $(\psi_{LOS}, \theta_{LOS})$ and $(\psi_{bLOS}, \theta_{bLOS})$ are derived from the image plane value $[y, z]$.

B. Derivation of LOS rate

There are five coordinate systems to derive the LOS rate: the inertial frame, body frame, strapdown seeker frame, pointing frame, and LOS frame. The strapdown seeker is fixed in the vehicle, so we can assume that the strapdown seeker frame and body frame are same. Each coordinate transformation matrix is summarized as follows (Won-Sang Ra and Ick-Ho Whang, 2002):

$$
C_{i}^{B} = C_{i}(\phi)C_{i}(\theta)C_{i}(\psi)
$$

$$
C_{b}^{p} = C_{p}(e_{x})C_{p}(e_{y})
$$

$$
C_{LOS}^{P} = C_{\phi_{LOS}}
$$

$$
C_{LOS}^{L} = C_{\lambda_{LOS}}
$$

Notations $e_{x}$ and $e_{y}$ are LOS angle error. The relations among the coordinate systems are illustrated in Figures 1, 5, and 6.
\[
\begin{align*}
\omega_\theta^b & = \begin{bmatrix} p & q & r \end{bmatrix}^T
\end{align*}
\]

(7)-a

\[
\begin{align*}
\omega_\theta^p & = \begin{bmatrix} p & q & r \end{bmatrix}^T
\end{align*}
\]

(7)-b

In the above equations, the superscript denotes the coordinate system in which the quantity is represented. By basic kinematics, the following equations hold.

\[
\begin{align*}
\omega_\theta^p - C_b^p \omega_\theta^b = \omega_\theta^p = \begin{bmatrix} 0 \\ \dot{e}_y \\ \dot{e}_z \end{bmatrix} + C_e \begin{bmatrix} 0 \\ e_y \\ e_z \end{bmatrix}
\end{align*}
\]

(8)-a

\[
\begin{align*}
\omega_\theta^p & = C_b^p \begin{bmatrix} p \\ q \\ r + \dot{e}_z \end{bmatrix} + \begin{bmatrix} e_y \\ e_y \\ e_z \end{bmatrix} + \begin{bmatrix} -pSe_yCe_z + qSe_yCe_z - (r + \dot{e}_z)Se_y \\ pSe_yCe_z + qSe_yCe_z + (r + \dot{e}_z)Ce_y \end{bmatrix} = \begin{bmatrix} p_p \\ q_p \\ r_p \end{bmatrix}
\end{align*}
\]

(8)-b

\[
\begin{align*}
\omega_\theta^{LOS} = C_1^{LOS} \begin{bmatrix} 0 \\ 0 \\ \dot{e}_y \end{bmatrix} + \begin{bmatrix} S\lambda & \dot{\lambda}_b \\ \dot{\lambda}_b \end{bmatrix} = \begin{bmatrix} S\lambda & \dot{\lambda}_b \\ \dot{\lambda}_b \end{bmatrix} + \begin{bmatrix} 0 \\ -\dot{\lambda}_b \end{bmatrix} = \begin{bmatrix} \lambda & -\dot{\lambda}_b \\ \dot{\lambda}_b \end{bmatrix}
\end{align*}
\]

(9)

\[
\begin{align*}
\dot{\lambda}_b = -C_{\phi_b} q_p - S_{\phi_b} r_p
\end{align*}
\]

(12)-a

\[
\begin{align*}
\dot{\lambda}_b = -\frac{S_{\phi}}{C\lambda_b} q_p + \frac{C_{\phi}}{C\lambda_b} r_p
\end{align*}
\]

(12)-b

where \(S_{\phi_b}\) and \(C_{\phi_b}\) can be derived by the following equations

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
1 \\
0 \\
1 \\
0 \\
1 \\
\end{bmatrix}
\]

(13)

**SIMULATION FOR LOS ANGLE AND RATE**

In this section, the performance of LOS angles and LOS rates are investigated in a simple homing missile system scenario with PNG law. The simulation flow chart is illustrated in Figure 7. The simulation package consists of a navigation division and a guidance control division. In the navigation division, the ATM_Sensor part senses the body’s acceleration and angular rate and image plane values. The ATM_PureINS part derives the vehicle’s position, attitude, LOS angle and LOS rate with respect to inertial frame. These data are applied to PNG law in the guidance control division.

The assumptions are as follows: the missile’s initial attitude is roll 0deg, pitch 20deg, yaw 0deg, and guidance law is PNG. The vehicle’s initial speed is 100m/s and Target speed is 10m/s. FOV limit is ±40deg and miss-distance threshold is within 10 meters.

The image plane value is the measurement of the strapdown seeker. Figure 8(a) shows the image plane values’ path which starts at a starting point and is terminated at a final point. From that value, we have derived LOS angles and LOS rates. In Figure 8(a), the value of starting point \([y,z]\) is [negative, negative], which means that the target is located in the down and right directions in body frame. At the initial point, the vehicle is launched at pitch 20deg attitude, so the LOS pitch angle with respect to body frame is -20deg and LOS pitch angle with respect to inertial frame is 0deg, as shown in Figure 8(b). At the initial time, the target is located at the north east direction, so the LOS yaw angles are +(plus) deg. At final time, the vehicle attacks the target within 10 meters, so LOS pitch and yaw with respect to body frame is approximately zero.
As time goes by, the LOS rate converges to zero according to PNG law. Figure 9(a) shows the top-down trajectory and Figure 9(b) shows the side trajectory of the vehicle.

The flight time is 10.02 seconds and miss-distance is 9.8575 meters. These simulation results mean that the derivation of LOS angles and LOS rates has significance which can be applied to PNG law.
STABILITY ANALYSIS OF STRAPDOWN SEEKER SCALE FACTOR ERROR

Two problems can be generated when guidance-control loop is implemented using the LOS angle from the strapdown seeker. The first problem is the trend that linearity errors and noises are proportional to the F.O.V. The second problem is as follows: because the strapdown seeker is rigidly fixed on the body, the output of strapdown seeker is not the input for the PNG law but rather the LOS angles represented in body frame. At this point, the output of the strapdown seeker is affected by the missile angular motion and this degrades the stability of the system.

If the strapdown seeker output is combined with the output of inertial sensors, the potential for instability exists if the scale factors and gain of the two sensors are not equal. The whole guidance-control system using the PNG law is depicted in Figure 10. This system is equipped with the strapdown seeker. The total transfer function of this system derived by Mason’s rule is illustrated in Equation 14. The vehicle’s equation and flight control system’s equations are illustrated in Equations 15 and 16.

\[
G(s) = \frac{N_y}{\lambda_y} = \frac{K_s s \frac{4V}{1845} \left( \begin{array}{c} N_y \\ N_y' \end{array} \right)}{1 + s \frac{\zeta}{\omega_n} \frac{4V}{1845} \left( \begin{array}{c} N_y \\ N_y' \end{array} \right) \left( \begin{array}{c} r_y \\ r_y' \end{array} \right) \frac{1}{s} (K_s - K_g)}
\]

(14)

\[
\left( \begin{array}{c} r_y \\ r_y' \end{array} \right) = \left( \begin{array}{c} r_y \\ y \end{array} \right) \left( \begin{array}{cc} 0.888 \frac{s}{0.7835} + 1 \\ \frac{s}{35.83} + 1 \\ \frac{s}{34.60} + 1 \end{array} \right)
\]

(15)

In Equation 14, if the strapdown seeker scale factor \( K_s \) and body gyro scale factor \( K_g \) are equal, two loops can be compensated as follows:

\[
G(s) = \frac{N_y}{\lambda_y} = \frac{s \frac{4V}{1845} \left( \begin{array}{c} N_y \\ N_y' \end{array} \right)}{1 + \frac{\zeta}{\omega_n} \frac{4V}{1845} \left( \begin{array}{c} N_y \\ N_y' \end{array} \right) \left( \begin{array}{c} r_y \\ r_y' \end{array} \right) \frac{1}{s} (K_s - K_g)}
\]

(17)

In this case, we performed the simulation. The miss-distance is 0.2913 meters and this value is the stable value when we assumed that the general miss-distance is 10 meters.

However, in the case of \( K_s > K_g \), the total transfer function became negative feedback; therefore, negative feedback occurred in the body angular rate, and the system can be unstable. In Figure 11, the LOS angle represented in the navigation frame is \( \lambda_y = \epsilon + \psi \).

After the scale factors discordance that is occurring, the LOS angle is changed to \( \lambda_y = \epsilon - \psi \) and system becomes unstable (Figure 12(b)).

![Figure 10. Guidance control system](image-url)
In addition, in the case of $K_s < K_g$, the total transfer function became positive feedback, therefore, positive feedback occurred in the strapdown seeker and system can be unstable (Figure 13). In Figure 11, the LOS angle represented in the body frame is $\psi = \dot{\psi} - \lambda$. After the scale factors discordance occurring, the LOS angle is changed to $\psi = \dot{\psi} + \lambda$ and system became unstable (Figure 13(b)).

In the case of discordance, the authors performed the 100 times Monte Carlo simulation to analyze the scale factor error. The assumption of IMU is 1mg accelerometer, 10deg/hr gyroscopes. When we add the 0.1% scale factor error to the YZ image plane, the mean of miss-distance is 0.2415 meters with a standard deviation of 0.0210 meters. Miss-distance of 100 times simulation is depicted in Figure 14(a) and ascending sort is depicted in Figure 14(b).

When we add the 0.5% scale factor error to the YZ image plane, the mean of the miss-distance is 1.1631 meters with a standard deviation of 0.6453 meters. Miss-distance of 100 times simulation is depicted in Figure 15(a) and ascending sort is depicted in Figure 15(b).
When we add the 1.0% scale factor error to the YZ image plane, the mean of miss-distance is 72.8280 meters with a standard deviation of 64.3471 meters. Miss-distance of 100 times simulation is depicted in Figure 16(a) and ascending sort is depicted in Figure 16(b). As the scale factor error increases, the miss-distance increases exceedingly.

The mean and standard deviation of miss-distance is illustrated in Table 1 and depicted in Figure 17 according to each scale factor error.

Table 1. Mean and standard deviation of miss-distance

<table>
<thead>
<tr>
<th>Scale factor error[%]</th>
<th>Mean[meters]</th>
<th>STD[meters]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10%</td>
<td>0.2415</td>
<td>0.021</td>
</tr>
<tr>
<td>0.20%</td>
<td>0.3522</td>
<td>0.1182</td>
</tr>
<tr>
<td>0.30%</td>
<td>0.6321</td>
<td>0.2388</td>
</tr>
<tr>
<td>0.40%</td>
<td>0.6473</td>
<td>0.1819</td>
</tr>
<tr>
<td>0.45%</td>
<td>0.8635</td>
<td>0.3087</td>
</tr>
<tr>
<td>0.47%</td>
<td>1.0137</td>
<td>0.4499</td>
</tr>
<tr>
<td>0.50%</td>
<td>1.1631</td>
<td>0.6453</td>
</tr>
<tr>
<td>0.53%</td>
<td>1.4255</td>
<td>0.8743</td>
</tr>
<tr>
<td>0.55%</td>
<td>1.8864</td>
<td>2.2968</td>
</tr>
<tr>
<td>0.60%</td>
<td>2.8001</td>
<td>3.5564</td>
</tr>
<tr>
<td>0.70%</td>
<td>10.9895</td>
<td>17.2225</td>
</tr>
<tr>
<td>0.80%</td>
<td>34.9645</td>
<td>38.2126</td>
</tr>
<tr>
<td>0.90%</td>
<td>51.7326</td>
<td>45.792</td>
</tr>
<tr>
<td>1.00%</td>
<td>72.8280</td>
<td>64.3471</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

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REFERENCES


http://www.globalsecurity.org/military/systems/munitions/apkws.htm

http://en.wikipedia.org/wiki/Advanced_Precision_Kill_Weapon_System

CONCLUSION

In this paper, we have presented the method of the LOS angle and LOS rate derivation from image plane values provided by strapdown seekers. Also we have analyzed the effects of strapdown seeker scale factor error.

The image plane provides a [y, z] value which is represented in the seeker frame. The seeker frame is the same as the body frame. To translate this value into the inertial frame value, we deal with a strapdown seeker structure and geometric relation between inertial and body frame. Simple but realistic simulations evaluate the method of LOS angle derivation. In order to derive the LOS rate in inertial frame, we handle 5 coordinate systems and calculate the translational connection between each coordinate system by the kinematics. Also presented is an application to the calculation of PNG law for missiles. PNG law needs the LOS rate in the inertial frame. The simulation for PNG law consists of two divisions which are the navigation division and guidance-control division. LOS angle and LOS rate are calculated in the navigation division and inputted into the PNG law in the guidance control division. The reasonable simulation demonstrates an appropriate result trajectory and shows that the method of LOS angle and LOS rate derivation is correct.

In order to analyze the effects of strapdown seeker scale factor error, we add the scale factor error gradually from 0.10% to 1.00%. When we assume that the requirement of attack is within 10 meters, in other words, the threshold of the miss-distance is within 10 meters, whole results of 100 times Monte Carlo simulation are satisfied with a 0.5% scale factor error. When we assume within 2 meters, whole results of 100 times Monte Carlo simulation are satisfied with a 0.45% scale factor error. We find that scale factor error affects the performance and stability of the guidance missile in a great measure. The further work towards a solution to this instability is the research of the dither adaptive method and the EKF method.